1. **Answer any four of the following:**

(a) **Define**

(i) Feasible solution to LPP.

(ii) Basic feasible solution to LPP.

(iii) Convex set.

Show that the set of all feasible solution to a LPP (if a feasible solution exists) is a convex set.

(b) If \( \{a_n\} \) is a non-increasing sequence of positive numbers and if \( \sum_{n=0}^{\infty} 2^n a_n \) converges, then show that \( \sum_{n=0}^{\infty} a_n \) converges.

(c) Determine components of velocity and acceleration of a moving particle along radial and transverse directions.

(d) Write a programme in C/recent computer language to evaluate the roots of a quadratic equation \( ax^2 + bx + c = 0 \) requesting the user to input the values of \( a, b, c \) and to output real roots root1 and root2.

(e) If \( \{f_n\} \) is a sequence of continuous real valued functions on the metric space \( X \) that converges uniformly to \( f \) on \( X \), then show that \( f \) is also continuous on \( X \).
SECTION - A

2. (a) Show that every group is isomorphic to a subgroup of a permutation group $A(S)$ for some appropriate $S$.  
State the name of this theorem.  
(b) If $\{v_1, v_2, \ldots, v_n\}$ is a basis of a vector space $V$ and if $\{w_1, w_2, \ldots, w_m\}$ is linearly independent in $V$, then show that $m \leq n$.  

3. (a) Define (i) Euclidean Domain.  
(ii) Principal Ideal Domain (PID).  
Show that every Euclidean Domain is PID.  
(b) State Cayley Hamilton theorem and using it find inverse of the matrix $A$ if it exists.  
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 4 \\ 3 & 4 & 8 \end{pmatrix}$$

SECTION - B

4. (a) If $f$ is a real-valued continuous function on a compact metric space $X$, then show that $f(X)$, range of $f$, is compact and $f$ attains a maximum and minimum at points of $X$.  
(b) Define absolute convergence and conditional convergence for improper integrals  
   of the type $\int_{a}^{\infty} f(x) \, dx$ for continuous function $f(x)$.  
   Show that $\int_{a}^{\infty} \frac{\sin x}{x} \, dx$ is convergent but not absolutely.  

5. (a) Define differentiability of a function of two variables at a point.  
Let $f : E \to \mathbb{R}$ be defined on a neighbourhood $E$ of $(a, b) \in \mathbb{R} \times \mathbb{R}$ such that $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ are continuous at $(a, b)$. Show that $f$ is differentiable at $(a, b)$. Is the converse of this is true? Justify your answer.
MNS

(b) Define a Riemann integral for a bounded real function on \([a, b]\).
Show that a bounded real function \(f\) is Riemann integrable on \([a, b]\) if and only if for every \(\varepsilon > 0\), there is a partition \(P\) of \([a, b]\) such that \(U(P, f) - L(P, f) < \varepsilon\).

SECTION - C

6. (a) Show that the general equation \(ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0\) of second degree represents a conic section.
(b) Define (i) Divergence of a vector point function.
    (ii) Curl of a vector point function.
(c) Show that \(\vec{b} \cdot \nabla(\vec{a} \cdot \nabla) = \frac{3(\vec{a} \cdot \vec{r})(\vec{b} \cdot \vec{r})}{r^3} - \frac{\vec{a} \cdot \vec{b}}{r^3}\)
(d) Show that \(\text{curl}(\vec{r} \times \vec{a}) = -2\vec{a}\).

7. (a) (i) Find the length of perpendicular from a point \((x_1, y_1, z_1)\) to the plane \(ax + by + cz = d\).
    (ii) Find equation of a sphere for which the circle \(x^2 + y^2 + z^2 + 2y - 2z + 2 = 0\), \(2x + 3y + 4z = 8\) is a great circle.
(b) Verify Gauss divergence theorem for \(\vec{f} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k\) taken over the rectangular parallelepiped \(0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c\).

SECTION - D

8. (a) (i) Explain when \(M(x, y) + N(x, y)y' = 0\) is said to be exact in some rectangle \(R\).
    (ii) Define integrating factor of the equation \(M(x, y)dx + N(x, y)dy = 0\), where \(M\) and \(N\) have continuous partial derivatives in some rectangle \(R\).
    Show that the equation \(M(x, y) + N(x, y)y' = 0\) is exact in \(R\) if and only if \(\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}\) in \(R\).
(b) Find the inverse of the coefficient matrix of the system.

\[
\begin{bmatrix}
1 & 1 & 1 \\
4 & 3 & -1 \\
3 & 5 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
6 \\
4
\end{bmatrix}
\]

by Gauss - Jordan method with partial pivoting and hence solve the system.

P.T.O.
9. (a) If \( \phi_1 \) is a solution of \( y'' + a_1(x)y' + a_2(x)y = 0 \) on an interval \( I \), and \( \phi_1(x) \neq 0 \) on \( I \), describe a method to determine a second linearly independent solution \( \phi_2 \) of this differential equation on \( I \).

Hence or otherwise find second linearly independent solution of

\[ y'' - 4xy' + (4x^2 - 2)y = 0 \]

after verifying that \( \phi_1(x) = e^{x^2} \) is a solution of this differential equation.

(b) Describe Trapezoidal and Simpson \( \frac{1}{3} \) rule to find \( \int_a^b f(x)dx \) numerically.

Find an approximate value of \( \int_0^1 \frac{dx}{1 + x} \) by using Trapezoidal and Simpson \( \frac{1}{3} \) rule and compare with exact solution.