महाराष्ट्र वनसेवा मुख्य परीक्षा - 2012

POO

2012

STATISTICS (Optional)

सांख्यिकी (वैकल्पिक)

100126

Time : 3 hours **Maximum Marks : 200**

Note :

(i) *Answers must be written in English.*

(ii) *Question No. 1 is* ***compulsory****. Of the remaining questions, attempt* ***any Four*** *selecting one question from* ***each section****.*

(iii) *Figure to the* ***RIGHT*** *indicates marks of the respective question.*

(iv) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(v) *Statistical and logarithmic tables will be supplied on request.*

(vi) *Use of your own simple electronic calculator is allowed.*

(vii) *Candidates should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise they will be penalised.*

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1. Answer *any four* of the following :

(a) Let X be a gamma random variable (r.v.) with shape parameter α and scale parameter λ. **10**

(i) Obtain the moment generating function (m.g.f.) of X.

(ii) Let Y be another gamma r.v. with shape parameter β and scale parameter λ and which is independent of X. Using the m.g.f. obtain the distribution of $X+Y$.

(b) In a CRD with 4 treatments, each having 5 replicates, obtain an expression for the residual sum of squares (R_0^2) and state, giving reasons, its distribution. **10**

(c) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the probability density function (p.d.f.) given by : **10**

$$f(x)=\begin{cases}\theta(1-x)^{\theta-1}, & \text{if } 0<x<1\\ 0, & \text{otherwise}\end{cases}$$

and $\theta > 1$. Find the maximum likelihood estimator (MLE) of θ.

(d) Construct the dual of the following Linear Programming (L.P.) primal problem. **10**

$$\begin{aligned}\text{Minimize } & z=5x_1+2x_2\\ \text{subject to } & x_1-x_2\geqslant 3\\ & 2x_1+3x_2\geqslant 5\\ & x_1, x_2\geqslant 0\end{aligned}$$

State one feasible solution for each problem. Are the solutions you have stated optimal for both the problems ?

(e) Let X_1, X_2, X_3, X_4, X_5 be independent normal r.v.s. with mean μ and variance σ^2. **10**

Let $U=\frac{1}{9}\{(2X_1-X_2-X_3)^2+(2X_2-X_1-X_3)^2+(2X_3-X_1-X_2)^2\}$

State, giving reasons, the distribution of $W=U/(X_4-X_5)^2$. Obtain 'a' such that $P[W > a]=0.05$

P.T.O.

SECTION - A

2. Answer the following sub-questions :

(a) Let $\{X_n, n \geqslant 1\}$ be a sequence of i.i.d. r.v.s. with the common m.g.f. M(t). Let N be a Poisson r.v. with mean λ. Let $S_N = X_1 + \dots + X_N$ and $S_o = 0$. Obtain the m.g.f. of S_N, when N is independent of $\{X_n\}$. **15**

(b) Suppose X is a Poisson r.v. with mean λ and Y is Binomial $(m, \lambda/m)$, and X and Y are independent. Derive a formula for $P[X=Y]$. **15**

(c) Let X be an exponential r.v. such that $P[X \leq 2] = 0.2$. **10**

(i) Obtain the mean of X.

(ii) Obtain $P[X > 2+S / X > S]$. Justify your answer.

3. Answer the following sub-questions :

(a) Let the joint density of (X, Y) be **15**

$$f(x, y) \left.\begin{matrix} = 2/(1+x+y)^3, & x>0, y>0 \\ =0 & \text{, otherwise} \end{matrix}\right.$$

(i) Obtain the marginal density of X.

(ii) Determine whether X and Y are independent.

(b) Let $X_1, X_2, \dots, X_{100}$ be independent Poisson r.v.s. with mean 4. Using the appropriate approximation and tables obtain : **15**

$P[X_1 + X_2 + \dots + X_{100} \leq 450]$

(c) Let X be a r.v. with p.d.f. $f(x) = \dfrac{e^{-x} \, x^n}{n!}$, $x > 0$, $f(x) = 0$, $x \leq 0$, and n a positive integer. Let Y be a Poisson r.v. with mean λ. Show that $P[X \geqslant \lambda] = P[Y \leq n]$. **10**

SECTION - B

4. Answer the following sub-questions :

(a) For a simple random sampling without replacement from a population of size N, let $\bar{y}$ denote the sample mean. Show that $N\bar{y}$ is an unbiased estimator of the population total. **15**

(b) In a RBD with 4 treatments and 5 blocks, derive a test of hypothesis of equality of any two treatment effects. **10**

(c) Let X_1 and X_2 be two independent exponential r.v.s. with mean $1/\lambda$ and let $U_1 = \min(X_1, X_2)$, $U_2 = \max(X_1, X_2)$. Show that the r.v.s. $V_1 = 2U_1$ and $V_2 = U_2 - U_1$ are independent and obtain their distribution. **15**

5. Answer the following sub-questions :

(a) In a stratified sampling with proportional allocation, consider a separate regression estimator (with known regression coefficient) of the population mean. Obtain the expected value and the variance of this estimator. 15

(b) Suppose a 2^3 factorial experiments with factors A, B, C, are to be run in two blocks with (ABC) confounded and the design is to be replicated four times. 10

(i) Write down the two blocks.

(ii) Write down the corresponding analysis of variance table.

(c) Let X_1, X_2, X_3 be independent r.v.s. with mean zero and variance 1, and let $S_n = X_1 + \dots + X_n$. Obtain the multiple correlation coefficient between S_3 and (S_1, S_2). Examine whether it is the same as the correlation between S_2 and S_3. 15

SECTION - C

6. Answer the following sub-questions :

(a) Let $X_1, X_2, \dots, X_n$ be a random sample from a Bernoulli distribution with $P[X_1 = 1] = P = 1 - P[X_1 = 0]$, $0 < P < 1$. 15

(i) Show that $X_1 + X_2 + \dots + X_n$ is a sufficient statistic for P.

(ii) Obtain an unbiased estimator for $P(1-P)$.

(iii) Is the estimator you have given in (ii) the only unbiased estimator for $P(1-P)$? Give reasons for your answer.

(b) Measurements of the diameters of a random sample of 400 ball bearings made by a certain machine showed a mean of 10.5 mm and a standard deviation of 0.8 mm. Find the 95% confidence limit for the mean diameter of the ball bearings. What should be the size of the sample if one wants to be 95% confident that the error of the estimate will not exceed 0.01 ? 10

(c) A sample of size 1 is taken from a distribution with p.d.f. 15

$$f(x) = \begin{cases} \frac{2}{\theta^2} \bullet (\theta - x), & \text{if } 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

Find the most powerful test of size $\alpha = 0.05$ of $H_0 : \theta = 1$ against $H_1 : \theta = 2$. State any major result you might use.

7. Answer the following sub-questions :

(a) Let X be an observation from a distribution with p.d.f. 15

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Where $\theta > 0$.

(i) Obtain the Cramér - Rao lower bounds corresponding to θ and θ^2.

(ii) Obtain an unbiased estimator of θ^2 and determine whether its variance attains the Cramér - Rao lower bound.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from uniform $(0, \theta)$ distribution. Obtain the maximum likelihood estimator of θ. Is the estimator unbiased for θ ? Give reasons for your answer. 10

(c) A company claims that the lifetime of a type of battery it manufactures is more than 250 hours. Measurement on lifetimes of 20 batteries of the company, selected randomly, are given below : 15

271	230	190	280	226
286	275	264	286	282
274	295	242	225	268
253	216	291	241	252

Using a distribution free test, determine whether the company's claim is justified at 0.05 significance level.

SECTION - D

8. Answer the following sub-questions :

(a) The following table gives the activities and the time required in a construction project. 10

Activity	Preceding activity	Time (days)
1 - 2	-	20
1 - 3	-	25
2 - 3	1 - 2	10
2 - 4	1 - 2	12
3 - 4	1 - 3, 2 - 3	5
4 - 5	2 - 4, 3 - 4	10

(i) Draw the activity network of the project.

(ii) Find the total float and the free-float for each activity. What do these indicate ?

(b) (i) Define : Laspeyres' Index number, Paasche's Index number and Fisher's Ideal Index. 15

(ii) Prove that Fisher's ideal index lies between Laspeyres' and Paasche's index numbers. What advantage does the Fisher ideal index have over the other two index numbers ?

(c) Samples of $n=6$ items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After 50 subgroups have been analyzed we have **15**

$$\sum_{i=1}^{50} \bar{x}_i = 1000 \quad \text{and} \quad \sum_{i=1}^{50} S_i = 75$$

(i) Compute the control limits fo- the $\bar{x}$ and S control charts.

(ii) Assume that all points on both charts plot within the control limits. What are the natural tolerance limits of the process ?

9. Answer the following sub-questions :

(a) The following table gives the cost in Rs. (thousands) of transportation of one item from a plant to a ware-house. **10**

Plant	Ware - House				Supply
	1	2	3	4	
P_1	3	4	9	2	23
P_2	6	5	8	8	27
Demand	12	13	15	10	

Obtain the initial feasible solutions using two different methods. Which method results in a lesser cost ?

(b) The following data gives the number (in lakhs) of cars produced for the years 1970 to 1979. **15**

Year	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
Cars	9.47	9.36	8.65	8.25	7.80	7.45	7.34	7.0	6.86	7.04

Obtain the trend values using :

(i) the method of semiaverages by taking the average as the mean;

(ii) the 5 - year moving average method.

State one disadvantage of the moving average method.

(c) Explain : **15**

(i) Single sampling plan,

(ii) Double sampling plan,

(iii) Operating characteristic (OC) curve.

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